A Singular Example in the Statistical Mechanics of a One-Dimensional System

M. L. Glasser¹

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To examine boundary effects in phase transitions, the following simple one-dimensional model has been considered. Let particles characterized by a continuous spin $-1 \le s_k \le 1$ be located at the points k = 0, 1, 2, ..., N on the line subject to the nearest-neighbor interaction

$$\mathscr{H}_{k,k'} = J \ln 2\pi (1 - |s_k s_{k'}|) \tag{1}$$

In studying the partition function of the system as a function of the spin at k = 0, which is held fixed, one analytic result has been found which is presented here as of possible intrinsic interest.

In the thermodynamic limit the partition function is

$$Z(s_0) = \lim_{N \to \infty} 2^N \int_0^1 ds_1 \cdots \int_0^1 ds_N \exp\left\{-\beta J(\ln 2\pi)N - \beta J \sum_{k=1}^\infty \ln\left(1 - s_{k-1}s_k\right)\right\}$$
(2)

where β is the inverse temperature. This quantity satisfies the singular integral equation

$$Z(s) = 2e^{-\beta J \ln 2\pi} \int_0^1 dt \, \frac{Z(t)}{(1-st)^{\beta J}} \tag{3}$$

¹ Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada.

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An exact solution to this equation has been found for only one nontrivial $case^{(1)}$: $\beta J = 1$, where

$$Z(s_0) \sim 2^N (1 - |s_0|)^{-1/2} \mathbf{K}(|s_0|^{1/2})$$
(4)

and **K** denotes the complete elliptic integral of the first kind. Since $Z(s_0)$ is singular at $s_0 = \pm 1$ we are led to the conjecture that this system undergoes an ordering transition when the end spin is held fixed at ± 1 .

REFERENCE

1. M. L. Glasser, SIAM Rev. (to be published).